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# COMMON FIXED POINT THEOREM FOR HYBRID PAIRS OF R-WEAKLY COMMUTING MAPPINGS

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**Abstract**. In this paper we established a common fixed point theorem for four mappings $f, g$ (crisp) and $S, T$ (fuzzy) of $R$ – weakly commuting mapping in a metric space.

## 1 Introduction

After the introduction of fuzzy sets by Zadeh [17], Butnariu [3], Chitra [5], Heilpern [6], Lee and Cho[9], Som [15], and others introduced the concept of fuzzy mappings and proved a fixed point theorem for fuzzy mappings. In 1975, Weiss [16], initiated the fixed point of fuzzy mappings. In 1987, Bose and Sahani [2] gave an improved version of Heilpern. In 2000, Arora and Sharma [1], proved a common fixed point theorem of fuzzy mappings satisfying a different inequality. Heilpern, Bose and Sahani and Arora and Sharma all considered fuzzy fixed point theorems in a linear metric space settings. In this series, recently Rashwan & Ahmed [14] proved a common fixed point theorem for a pair of fuzzy mappings.

A fuzzy function is a generalization of the concept of classical function. A classical function $f$ is a correspondence from the domain $D$ of definition of the function $f$ into a space $S$; $f(D) \subseteq S$ is called the range of $f$. Different features of the classical concepts of a function can be considered to be fuzzy rather than crisp. Therefore, different degrees of fuzzification of the classical notion of a function are conceivable.

(1) There can be a crisp mapping from a fuzzy set, which carries along the fuzziness of the domain and therefore generates a fuzzy set. The image of a crisp argument would again be crisp.

(2) The mapping itself can be fuzzy, thus blurring the image of a crisp argument. This we shall call a *fuzzy function* or *fuzzy mapping*.

(3) Ordinary functions can have fuzzy properties or be contained by fuzzy constraints.

In this paper first the coincidence point of a crisp mapping and a fuzzy mapping has been defined. Then $R$ – weakly commutativity is introduced for a pair of crisp

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mapping & a fuzzy mapping (see [7, 8, 12, 13]). At last, a common coincidence points theorem has been proved for the combinations of crisp mappings & fuzzy mappings together using the notion of $R-$ weakly commuting mappings.

## 2 Preliminaries

Here we cite briefly some definitions, lemmas and propositions noted in [14]. Let $(X, d)$ be a metric linear space. A fuzzy set in $X$ is a function with domain $X$ and values in $[0, 1]$. If $A$ is a fuzzy set and $x \in X$, then the function values $A(x)$ is called the grade of membership of $x$ in $A$. The $\alpha$-level set of $A$ denoted by $A_\alpha$, is defined by

$$A_\alpha = \{x : A(x) \geq \alpha \text{ if } \alpha \in (0, 1]\},$$
$$A_0 = \overline{\{x : A(x) > 0\}}$$

where $\overline{B}$ denotes the closure of the set $B$.

**Definition 1.** *A fuzzy set $A$ in $X$ is said to be an approximate quantity iff $A$ is compact and convex in $X$ for each $\alpha \in [0, 1]$ and $\sup_{x\in X} A(x) = 1$. Let $F(X)$ be the collection of all fuzzy sets in $X$ and $W(X)$ be a sub-collection of all approximate quantities.*

**Definition 2.** *Let $A, B \in W(X)$, $\alpha \in [0, 1]$. Then*

$$p_\alpha(A, B) = \inf_{x\in A_\alpha, y\in B_\alpha} d(x, y),$$

$$\delta_\alpha(A, B) = \sup_{x\in A_\alpha, y\in B_\alpha} d(x, y)$$

$$D_\alpha(A, B) = H(A_\alpha, B_\alpha) = \max\{\sup_{a\in A_\alpha} d(a, B_\alpha), \sup_{b\in B_\alpha} d(A_\alpha, b)\}$$

*where $H$ is the Hausdorff distance and $D$ is called generalized Hausdorff distance or metric in the collection $CP(X)$ of all non empty compact subsets of $X$. Also for $CB(X)$, set of non-empty closed subset of $X$, as follows:*

$$p(A, B) = \sup_\alpha p_\alpha(A, B),$$

$$\delta(A, B) = \sup_\alpha \delta_\alpha(A, B)$$

*and*

$$D(A, B) = \sup_\alpha D_\alpha(A, B)$$

*It is noted that $p_\alpha$ is non-decreasing function of $\alpha$ and thus $p(A, B) = p_1(A, B)$. In particular if $A = \{x\}$, $p(\{x\}, B) = p_1(x, B) = d(x, B_1)$.*

**Definition 3.** *Let $A, B \in W(X)$. Then $A$ is said to be more accurate than $B$ (or $B$ includes $A$), denoted by $A \subset B$ iff $A(x) \leq B(x)$ for each $x \in X$.*

*Let $X$ be an arbitrary set and $Y$ be any linear metric space. $F$ is called a fuzzy mapping iff $F$ is a mapping from the set $X$ into $W(Y)$ with membership function $F(x)(y)$. The function value $F(x)(y)$ is the grade of membership of $y$ in $F(x)$.*

**Lemma 4.** *([6]) Let $x \in X$. $A \in W(X)$ and $\{x\}$ be a fuzzy set with membership function equal to characteristic function of the set $\{x\}$. Then $x \subset A$ if and only if $p_\alpha(x, A) = 0$ for each $\alpha \in [0, 1]$.*

**Lemma 5.** *([6]) $p_\alpha(x, A) \leq d(x, y) + p_\alpha(y, A)$ for any $x, y \in X$.*

**Lemma 6.** *([6]) If $\{x_0\} \subset A$ then $p_\alpha(x_0, B) \leq D_\alpha(A, B)$ for each $B \in W(X)$.*

**Proposition 7.** *([9]) Let $(X, d)$ be a complete metric linear space and $F : X \to W(X)$ be a fuzzy mapping and $x_0 \in X$. Then there exists $x_1 \in X$ such that $x_1 \in F(x_0)$.*

**Remark 8.** *Let $J : X \to X$ and $F : X \to W(X)$ such that $\cup\{F(X)\}_\alpha \subseteq J(X)$ for each $\alpha \in [0, 1]$. Suppose $J(X)$ is complete. Then, by an application of Proposition 2.1, it can be easily shown that for any chosen point $x_0 \in X$ there exists a point $x_1 \in X$ such that $\{J(x_1)\} \subseteq F(x_0)$.*

**Proposition 9.** *([10]) If $A, B \in CP(X)$, a collection of all nonempty compact subset i.e. $A, B \in CP(X)$ subset of $X$ and $a \in A$, then there exists $b \in B$ such that*

$$d(a, b) \leq H(A, B)$$

Recently Rashwan and Ahmad [14] introduced the set $G$ of all continuous functions $g : [0, \infty)^5 \to [0, \infty)$ with the following properties :

(i) $g$ is non decreasing in $2^{nd}$, $3^{rd}$, $4^{th}$ and $5^{th}$ variables.

(ii) If $u, v \in [0, \infty)$ are such that $u \leq g(v, v, u, u + v, 0)$ or $u \leq g(v, u, v, 0, u + v)$ then $u \leq hv$ where $0 < h < 1$ is a given constant.

(iii) If $u \in [0, \infty)$ is such that $u \leq g(u, 0, 0, u, u)$ then $u = 0$.

Then Rashwan and Ahmad proved the following theorem:

**Theorem 10.** *Let $X$ be a complete metric linear space and let $F_1$ and $F_2$ be fuzzy mappings from $X$ into $W(X)$. If there is a $g \in G$ such that for all $x, y \in X$*

$$D(F_1(x), F_2(y)) \leq g[d(x, y), p(x, F_1(x)), p(y, F_2(y)), p(x, F_2(y)), p(y, F_1(x)]$$

*then there exists $z \in X$ such that $\{z\} \subseteq F_1(z)$ and $z \subseteq F_2(z)$.*

# 3 Main result

First of all, we introduce the following definitions and examples:

**Definition 11.** *Let $I : X \to X$ be a self mapping and $F : X \to W(X)$ a fuzzy mapping. Then a point $u \in X$ is said to be coincidence point of $I$ and $F$. If $\{I(u)\} \subset F(u)$ i.e. $I(u) \in \{F(u)\}_1$.*

**Definition 12.** *The mappings $I : X \to X$ and $F : X \to W(X)$ are said to be R-weakly commuting if for all $x$ in $X$, $I\{F(x)\}_\alpha \in CP(X)$ and there exists a positive number $R$ such that*

$$H(I\{Fx\}_\alpha, \{FIx\}_\alpha) \leq Rd(Ix, \{Fx\}_\alpha),$$

*for all $\alpha \in [0,1]$, where*

$$\{Fx\}_\alpha = \{y \in X | F(x)(y) \geq \alpha\}.$$

**Example 13.** *Let $(X,d)$ be a metric space where $X = [0,1]$ and $d$ denote the usual metric. Define the mapping $I : X \to X$ such that $Ix = \frac{x}{2}$ for all $x \in X$ and $F : X \to W(X)$ a fuzzy mapping such that for all $x \in [0,1]$, $Fx$ is a fuzzy set on $X$ given by, for all $x, y \in [0,1]$,*

$$F(x)(y) = \begin{cases} 0 & \text{if } 0 \leqslant y < \frac{x+1}{2} \\ \frac{y-x}{1-x} & \text{if } \frac{x+1}{2} \leqslant y \leqslant 1 \end{cases}$$

*when $0 \leq \alpha < \frac{1}{2}$ then*

$$\{Fx\}_\alpha = \left[\tfrac{1}{2}(1+x), 1\right], \ \{Ix\} = \left\{\frac{x}{2}\right\},$$

$$\{FIx\}_\alpha = \left(F\left(\frac{x}{2}\right)\right)_\alpha = \left[\tfrac{1}{2} \ \left(1+\tfrac{x}{2}\right), 1\right] \ , \ I\{Fx\}_\alpha = \left[\frac{1}{4}(1+x), \frac{1}{2}\right]$$

*and so,*

$$H(I\{Fx\}_\alpha, \{FIx\}_\alpha) = \left\{\left|\tfrac{1}{2}\left(1+\tfrac{x}{2}\right) - \tfrac{1}{4}(1+x)\right|, \left|1-\tfrac{1}{2}\right|\right\} = \frac{1}{2},$$

*and*

$$d(Ix, \{Fx\}_\alpha) = \tfrac{1}{2}$$

*when $\frac{1}{2} \leq \alpha \leq 1$, then $\{Fx\}_\alpha = [x + (1-x), 1]$,*

$$I\{Fx\}_\alpha = \left[\tfrac{x}{2} + \tfrac{\alpha}{2}(1-x), \tfrac{1}{2}\right]$$

*and*

$$\{FIx\}_\alpha = \left[\tfrac{x}{2} + \alpha\left(1 - \tfrac{x}{2}\right),\ 1\right].$$

*Now we have,*

$$H(I\{Fx\}_\alpha, \{FIx\}_\alpha) = \max\ \left\{\left|\ \left(\tfrac{x}{2}\ +\ \alpha\ \left(1 - \tfrac{x}{2}\right)\right)\ -\ \left(\tfrac{x}{2}\ +\ \tfrac{\alpha}{2}\ (1 - x)\right)\ \right|,\ \tfrac{1}{2}\right\}$$
$$= \max\left\{\tfrac{\alpha}{2},\ \tfrac{1}{2}\right\} = \frac{1}{2}$$

*and*

$$d(Ix, \{Fx\}_\alpha) = \tfrac{x}{2} + \alpha\,(1 - x)\ \geqslant\ \tfrac{x}{2}\ +\ \tfrac{1}{2}\,(1\ -\ x) = \tfrac{1}{2},\ \textit{where}\ \frac{1}{2} \leqslant \alpha \leqslant 1$$

*Hence for* $R = 1$*, we have* $H(I\{Fx\}_\alpha, \{FIx\}_\alpha) \leq Rd(Ix, \{Fx\}_\alpha)$ *and so* $F, I$ *are* $R$*-weakly commuting.*

**Example 14.** *Let* $I : X \to X$ *be such that*

$$Ix = \left\{\frac{x}{2}\right\}$$

*and* $F : X \to W(X)$ *be defined as*

$$F(x)(y) = \begin{cases} 0 & \text{if } 0\ \leqslant\ y \text{ ¡ } x \\ \frac{y-x}{1-x} & \text{if } x\ \leqslant\ y\ \leqslant\ 1 \end{cases}$$

*Now, we have for all* $0 \leq \alpha \leq 1$*,*

$$\{Fx\}_\alpha = [x + (1 - x), 1],$$

$$I\{Fx\}_\alpha = \left[\tfrac{x}{2}\ +\ \tfrac{\alpha}{2}\ (1 - x),\ \tfrac{1}{2}\right]$$

*and*

$$\{FIx\}_\alpha = \left[\tfrac{x}{2}\ +\ \alpha\ \left(1 - \tfrac{x}{2}\right),\ 1\right].$$

*Then, similarly we get*

$$H(I\{Fx\}_\alpha, \{FIx\}_\alpha) = \tfrac{1}{2}.$$

*But* $d(Ix, \{Fx\}_\alpha) = \frac{x}{2} + (1 - x)$*, which can be made as small as possible by taking* $\alpha$ *and* $x$ *very small. Thus no* $R > 0$ *can serve the purpose. Hence* $F$ *and* $I$ *are not* $R$ *- weakly commuting.*

We prove the following theorem:

**Theorem 15.** *Let* $I, J$ *be mappings of a metric space* $X$ *into itself and let* $F_1, F_2 : X \to W(X)$ *be fuzzy mappings. Let* $G$ *be the set of all continuous functions* $g : [0, \infty)^5 \to [0, \infty)$ *with the following properties:*

*(i)* $g$ *is non decreasing in* $2^{nd}$*,* $3^{rd}$*,* $4^{th}$ *and* $5^{th}$ *variables;*

*(ii) If* $u, v \in [0, \infty)$ *are such that* $u \leq g(v, v, u, u+v, 0)$ *or* $u \leq g(v, u, v, 0, u+v)$ *then* $u \leq hv$ *where* $0 < h < 1$ *is a given constant.*

*(iii) If* $u \in [0, \infty)$ *is such that* $u \leq g(u, 0, 0, u, u)$ *then* $u = 0$.

*(iv) (a)* $\cup \{F_1 X\}_\alpha \subset J(X)$

*(b)* $\{F_2 X\}_\alpha \subset I(X)$ *for each* $\alpha \in [0, 1]$,

*(v) suppose there is a* $g \in G$ *such that for all* $x, y \in X$, *and* $I, J, F_1$ *and* $F_2$ *satisfy the following conditions:*

$$D(F_1 x, F_2 y) \leq g[d(Ix, Jy), p(Ix, F_1 x), p(Jy, F_2 y), p(Ix, F_2 y), p(Jy, F_1 x)]$$

*and*

*(vi) the pairs* $F_1$, $I$ *and* $F_2, J$ *are* $R$ *- weakly commuting. Suppose that one of* $I(X)$ *or* $J(X)$ *is complete, then there exists* $z \in X$ *such that* $Iz \subseteq F_1 z$ *and* $Jz \subseteq F_2 z$.

*Proof.* Let $x_0 \in X$ and suppose that $J(X)$ is complete. Taking $y_0 = Ix_0$ by Remark 8, and (a) in (iv) there exist points $x_1, y_1 \in X$ such that $\{y_1\} = Jx_1 \subseteq F_1 x_0$. For this point $y_1$, by Proposition 7, there exists a point $y_2 \in \{F_2 x_1\}_1$. But, by (b) in (iv) there exists $x_2 \in X$ such that $\{y_2\} = \{Ix_2\} \subseteq F_2 x_1$. Now by Proposition 9 and condition (v), we obtain

$$\begin{aligned} d(y_1, y_2) &\leq D_1(F_1 x_0, F_2 x_1) \leq D(F_1 x_0, F_2 x_1) \\ &\leq g[d(Ix_0, Jx_1), p(Ix_0, F_1 x_0), p(Jx_1, F_2 x_1), p(Ix_0, F_2 x_1), p(Jx_1, F_1 x_0)] \\ &\leq g[d(y_0, y_1), d(y_0, y_1), d(y_1, y_2), d(y_0, y_1) + d(y_1, y_2, ), 0]. \end{aligned}$$

which, by (ii) gives

$$d(y_1, y_2) \leq hd(y_0, y_1).$$

Since $\{F_2 x_1\}_1$, $\{F_1 x_2\}_1 \in CP(X)$ and $y_2 = Ix_2 \in \{F_2 x_1\}_1$ therefore, by proposition 2.2, there exists $y_3 \in \{F_1 x_2\}_1 \subseteq J(X)$ and hence there exists $x_3 \in X$ such that $\{y_3\} = \{Jx_3\} \subseteq \{F_1 x_2\}_1$. Again

$$d(y_2, y_3) \leq hd(y_1, y_2).$$

Thus, by repeating application of Proposition 9, and (a) ,(b) in (iv), we construct a sequence $y_k$ in $X$ such that, for each $k = 0, 1, 2, ......$

$$\{y_{2k+1}\} = \{Jx_{2k+1}\} \subseteq F_1(x_{2k})$$

and

$$\{y_{2k+2}\} = \{Ix_{2k+2}\} \subseteq F_2(x_{2k+1}).$$

and $d(y_k, y_{k+1}) \leq hd(y_{k+1}, y_k)$. Then, as in proof of Theorem 15, in [3], the sequence $y_k$, and hence any subsequence thereof, is Cauchy. Since $J(X)$ is complete then $Jx_{2k+1} \to z = Jv$ for some $v \in X$. Then

$$d(Ix_{2k}, Jv) \leq d(Ix_{2k}, Jx_{2k+1}) + d(Jx_{2k+1}, Jv) \to 0 \text{ as } k \to \infty.$$

Hence $Ix_{2k} \to Jv$ as $k \to \infty$.

Now, by Lemma 5, Lemma 6, and condition (v)

$$\begin{aligned}p(z, F_2v) &\leq d(z, Jx_{2k+1}) + D(F_1x_{2k}, F_2v)\\ &\leq d(z, Jx_{2k+1}) + g[d(Ix_{2k}, Jv), p(Ix_{2k}, F_1x_{2k}), p(Jv, F_2v), p(Ix_{2k}, F_2v), p(Jv, F_1x_{2k})]\\ &\leq d(z, Jx_{2k+1}) + g[d(Ix_{2k}, z), p(y_{2k}, y_{2k+1}), p(z, F_2v), p(Ix_{2k}, F_2v), d(z, y_{2k+1})]\end{aligned}$$

letting $k \to \infty$ it implies,

$$p(z, F_2v) \leq g(0, 0, p(z, F_2v), p(z, F_2v), 0)$$

which, by (ii), yields that $p(z, F_2v) = 0$. So by Lemma 4, we get $\{z\} \subseteq F_2v$ i.e. $Jv \in \{F_2v\}_1$. Since by (iv)(b), $\{F_2(X)\}_1 \subseteq I(X)$ and $Jv \in \{F_2v\}_1$ therefore there is a point $u \in X$ such that

$$Iu = Jv = z \in \{F_2v\}_1.$$

Now, by Lemma 6, we have

$$\begin{aligned}p(Iu, F_1u) = p(F_1u, Iu) &\leq D_1(F_1u, F_2v) \leq D(F_1u, F_2v)\\ &\leq g[d(Iu, Jv), p(Iu, F_1u), p(Jv, F_2v), p(Iu, F_2v), p(Jv, F_1u)]\end{aligned}$$

yielding thereby

$$p(Iu, F_1u) \leq g[0, p(Iu, F_1u), 0, 0, p(Iu, F_1u)]$$

which, by (ii), gives $p(Iu, F_1u) = 0$. Thus, by Lemma 4, $Iu \subseteq F_1u$, i.e. $Iu \in \{F_1u\}_1$.

Now, by $R$-weakly commutativity of pairs $F_1, I$ and $F_2, J$, we have

$$H(I\{F_1u\}_1, \{F_1\ Iu\}_1) \leq Rd(Iu, \{F_1u\}_1) = 0$$

$$H(J\{F_2v\}_1, \{F_2Jv\}_1) \leq Rd(Jv, \{F_2v\}_1) = 0$$

which gives $I\{F_1u\}_1 = \{F_1Iu\}_1 = \{F_1z\}$, and $\{JF_1v\}_1 = \{F_2Jv\}_1 = \{F_2z\}_1$ respectively.

But $Iu \in \{F_1u\}_1$ and $Jv \in \{F_2v\}_1$ implies

$$Iz = IIu \in I\{F_1u\}_1 = \{F_1z\}_1$$

$$Jz = JJv \in J\{F_2v\}_1 = \{F_2z\}_1.$$

Hence $Iz \subseteq F_1z$ and $Jz \subseteq F_2z$. Thus the theorem completes. □

**Remark 16.** *If $J(X)$ is complete, then in Theorem 15, it is sufficient that (iv)(b) holds only for $\alpha = 1$, because it becomes crisp set. Similarly if $I(X)$ is complete then (iv)(a) holds for $\alpha = 1$, is sufficient to consider.*

**Corollary 17.** *If taking $I = J =$identity in Theorem 15, we get easily Theorem 10.*

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