



ON THE PERSISTENCE PROPERTIES OF THE CROSS-COUPLED CAMASSA-HOLM SYSTEM

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Abstract. In this paper we examine the evolution of solutions, of a recently-derived system of cross-coupled Camassa-Holm equations, that initially have compact support. The analytical methods which we employ provide a full picture for the persistence of compact support for the momenta. For the solutions of the system itself, the answer is more convoluted, and we determine when the compactness of the support is lost, replaced instead by an exponential decay rate.

1. Introduction

This paper is concerned with the persistence of compact support in solutions to a recently derived cross-coupled Camassa-Holm (CCCH) equation [7], which is given by

$$m_t + 2v_x m + v m_x = 0, \quad n_t + 2u_x n + u n_x = 0 \quad (1)$$

where $m = u - u_{xx}$ and $n = v - v_{xx}$. This system generalises the celebrated Camassa-Holm (CH) equation [1], since for $u = v$ the system (1) reduces to two copies of the CH equation

$$m_t + 2u_x m + u m_x = 0.$$

The CH equation models a variety of phenomena, including the propagation of uni-directional shallow water waves over a flat bed [1, 8, 12, 16, 17]. The CH equation possesses a very rich structure, being an integrable infinite-dimensional Hamiltonian system with a bi-Hamiltonian structure and an infinitely many conservation laws [1, 4, 15]. It also has a geometric interpretation as a re-expression of the geodesic flow on the diffeomorphism group of the circle [14]. One of the most interesting features of the CH equation, perhaps, is the rich variety of solutions it admits. Some solutions exist globally, whereas others exist only for a finite length of time, modelling wave breaking [3, 6].